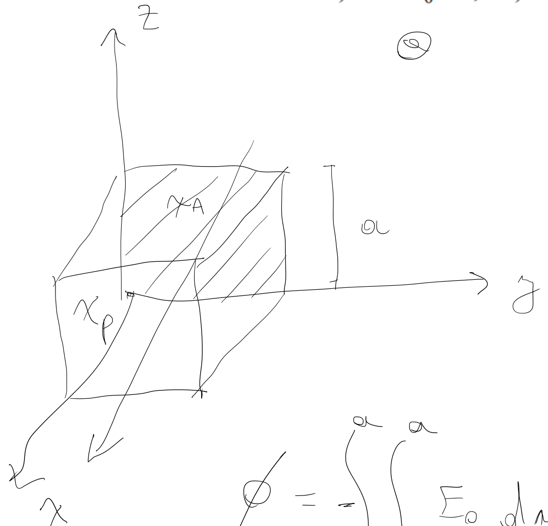
$l = a$

11. Un cubo de lado a tiene sus aristas paralelas a los ejes cartesianos y uno de sus vértices se encuentra en el origen de coordenadas. Hallar el flujo del campo eléctrico a través de su superficie, la densidad de carga y la carga total encerrada si:

a) $\vec{E} = E_0\,\breve{x}$; b) $\vec{E} = E_0\,x\,\breve{x}$; c) $\vec{E} = E_0\,x^2\,\breve{x}$; d) $\vec{E} = E_0\left(y\,\breve{x} + x\,\breve{y}\right)$



a)

$$\phi_E = \oint_S \bar{E}\cdot d\bar{S} = \iint_{x_A} E_0 \underbrace{\hat{x}\cdot(-\hat{x})}_{-1}\, dy\, dz + \iint_{x_p} E_0 \underbrace{\hat{x}\cdot\hat{x}}_{1}\, dy\, dz + \cancel{\iint_{y_A} E_0 \underbrace{\hat{x}\cdot\hat{y}}_{0}\, dz\, dx + \dots}$$

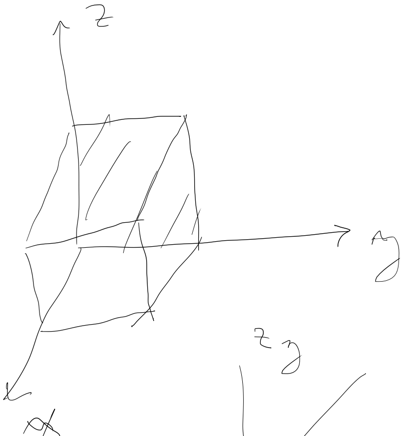
$$\phi = -\int_0^a\!\!\int_0^a E_0\, dy\, dz + \int_0^a\!\!\int_0^a E_0\, dy\, dz = 0$$

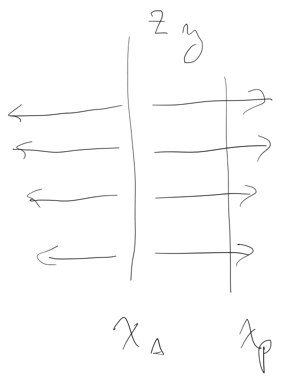
$$q_{enc} = \varepsilon_0 \oint \bar{E}\cdot d\bar{S} = \varepsilon_0\, \phi_E = 0$$

$$\rho = \varepsilon_0\, \bar{\nabla}\cdot\bar{E} = \varepsilon_0 \left(\frac{\partial E_0}{\partial x} + \frac{\partial 0}{\partial y} + \frac{\partial 0}{\partial z} \right)$$

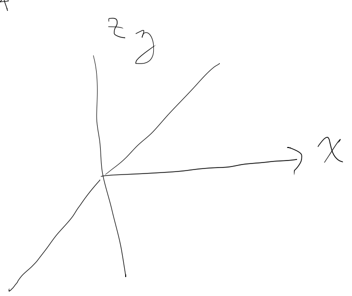
$$\rho = \varepsilon_0 \cdot 0 = 0$$

b) $\bar{E} = E_0\, x\, \hat{x}$





$$\phi_E = \oint_S \bar{E}\cdot d\bar{S} = \cancel{\iint_{x_A} E_0\, x\, \hat{x}\cdot(-\hat{x})\, dy\, dz}^{\,x_A = 0\ \to\ 0} + \iint_{x_p} E_0\, x \underbrace{\hat{x}\cdot\hat{x}}_{1}\, dy\, dz \quad (x_p = a)$$



$$\phi_E = \int_0^a\!\!\int_0^a E_0\, a\, dy\, dz = E_0\, a\, a^2 = \boxed{E_0\, a^3}$$

$$\frac{dq}{dV} = \rho$$

$$q_{enc} = \varepsilon_0\, \phi_E = \varepsilon_0 E_0 a^3$$

$$q = \iiint_V \rho\, dV$$

$$\rho = \varepsilon_0\, \bar{\nabla}\cdot\bar{E} = \varepsilon_0 \left(\frac{\partial (E_0 x)}{\partial x} + \frac{\partial 0}{\partial y} + \frac{\partial 0}{\partial z} \right) = \varepsilon_0 E_0$$

$$\boxed{\rho = \varepsilon_0 E_0}$$

$$q = \rho \cdot a^3 = \varepsilon_0 E_0 a^3$$